Combining finite element analyses and mechanical models for the assessment of reinforced concrete slabs

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Abstract

The influence of the level of deformation of the flexural reinforcement on the punching strength is well-established and mechanically founded since the pioneer works of Kinnunen and Nylander in the 1960's. In code provisions, this influence is sometimes reflected by means of indirect parameters such as the flexural reinforcement ratio (as in many empirical formulae) or by means of mechanical approaches accounting for the ratio between the applied load and the flexural resistance of the specimen (as in *fib*'s MC2010). An accurate estimate of this parameter (level of deformation of the flexural reinforcement) is nevertheless complex in many situations, due to the determination of the moment field and the potential redistributions of bending moments. In order to provide a consistent approach to determine the punching strength, it has been recently proposed to couple refined finite element (FE) models describing accurately the flexural behavior of slabs (accounting for the influence of cracking, yielding and moment redistributions) with a robust strain-based failure criterion in shear. In this framework, the goal of this paper is to review specific case studies in which redistributions after cracking in bending and shear may be governing on the overall behavior of reinforced concrete slabs. Load–displacement responses of the investigated case studies are evaluated by means of FE analyses using a multi-layered shell formulation. Eventually, the numerical global response is combined with the failure criterion of the Critical Shear Crack Theory, that can directly consider the influence of the flexural response on the punching or shear capacity.

1 Introduction

The use of reinforced concrete flat slabs is widely accepted as an economical and flexible solution in construction. Despite their redundancy and robustnbess in bending, the slab–column connections are particularly sensitive regions where stress concentrations occur due to the interaction between flexural and shear deformations and may potentially lead to brittle punching failures. Since the pioneer works of Kinnunen and Nylander [1] in the 1960's, the influence of the flexural reinforcement deformation on the punching strength is well-established and mechanically validated by experimental tests, see Figure 1a. Significant efforts have been done on the behavior of interior and edge slab – column connections for establishing theoretical approaches, such as the works by Moe [4], Broms [5], Mast [6], Moehle [7] as well as experimental works performed, for instance, by Kinnunen [8], Stamenkovic and Chapman, [9], Regan [10].

Despite the fact that many design provisions for punching are still based on empirical formulas [2] and account for the interaction between the flexural deformation and the shear strength in an indirect manner (by means for instance of the consideration of the reinforcement ratio), mechanical approaches (as in *fib*'s MC2010, [3]) have also been developed accounting explicitely for such relation. In both cases, design provisions are still based on experimental results on isolated test specimens simulating the region of negative bending moment in a continuous slab (i.e. the slab sector delimited by the line of moment contraflexure at a distance r_s from the axis of the column, Figure 1b). Although simple to perform, these test results might not describe the actual level of deformation of reinforced concrete slabs whose behavior may be affected by cracking, potential bending and shear redistributions due to slab continuity and confinement effects (more details on these effects, Figure 1b-c, can be consulted elsewhere, [12]). Moreover, such approaches consider in a simplified manner geometrical and loading asymmetries, typical of actual reinforced concrete members in which, for instance, gravity loads might be coupled with horizontal forces, such as seismic or wind actions. This is the case, for instance, of edge slab – column connections in which the overall behavior is not only associated to a shear force V, but also to an unbalanced moment M between the column and the slab. Therefore, uneven distribution of bending moments around the control perimeter may lead to an increase of crack openings accompanied by a likely reduction of the punching resistance [13], [14].

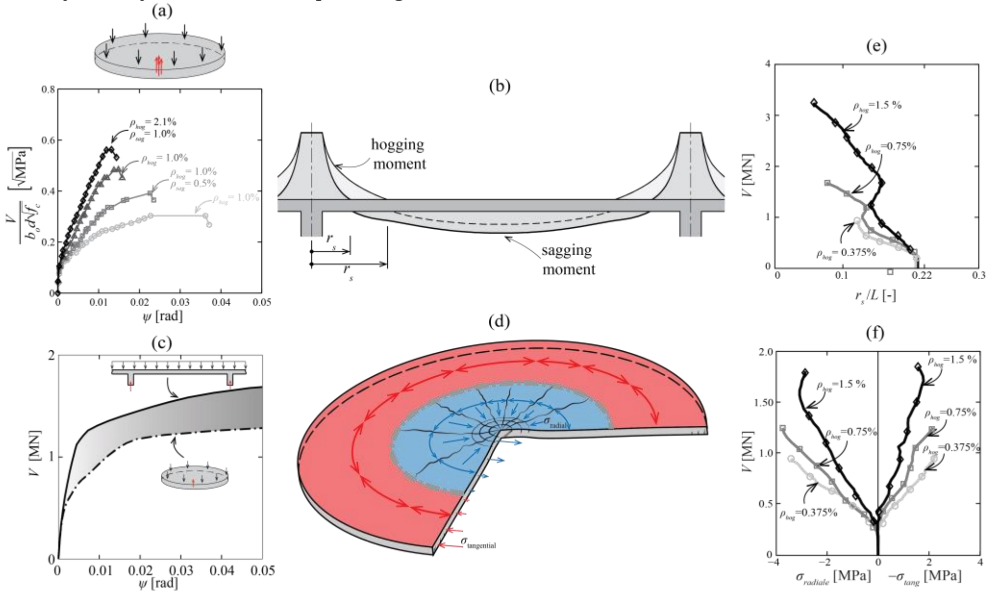

Fig. 1 (a) Influence of flexural reinforcement on the ultimate punching capacity of isolated specimens, [1]; (b) Redistributions of moments between hogging and sagging region; (c) Comparison of load-rotation response between an isolated and a continuous member; (d) cracking pattern of a continuous slab and formation of compressive (blue)/tensile (red) membrane forces; (e) redistributions of radial bending moments for different hogging reinforcement ratios in an inner column; (f) radial and tangential forces around the column as a function of hogging reinforcement ratio (adapted from Cantone et al. (2018), [11]).

2 Mechanical behavior of reinforced concrete slabs

As shown recently by several researchers with numerical and experimental investigations [12], [15], [16], [10], the overall behavior of reinforced concrete slabs at ultimate limit state may deviate from the response of isolated specimens adopted to calibrate actual code provisions. As briefly mentioned in the previous paragraph, several phenomena may influence the flexural behavior of reinforced concrete slabs:

(1) Mid-span reinforcement might play an important role in the redistribution of bending moments between hogging and sagging area [12], [15].

(2) In isolated specimens, flexural cracking allows the member to expand laterally. This might not be the case of continuous members in which self-confinement or external confinement provided by rigid elements may constrain the lateral expansion required by the opening of flexural cracks.

(3) In the case of edge slab-column connections, unbalanced moments trigger concentrations of shear forces leading to larger crack openings

(4) Moment redistributions due to flexural cracking may influence load eccentricity, Figure 2, at the edge slab – column connection.

The approach of the Critical Shear Crack Theory (CSCT) [17], theoretical ground of the punching provisions of *fib*'s Model Code 2010, has been adopted for the evaluation of the ultimate punching capacity. According to Kinnunen and Nylander's observation that the punching capacity is related to flexural deformations [1], Muttoni proposed a failure criterion accounting for the crack opening, assumed to be proportional to the parameter $\psi \cdot d$ (rotation of the member times its effective depth). In addition, the hyperbolic failure criterion accounts for strain and size effects in a consistent manner considering the roughness of the shear crack, the effective depth and the column size, [18], [19]. As proposed by Muttoni (2008) [17], and acknowledged by MC2010, the failure criterion may be combined with a reasonable load-rotation relationship describing the flexural behavior, see Figure 2. Following this approach, the best accuracy on the punching strength is obtained for the most realistic estimates of the flexural response of the slab. Next chapter will introduce the fundamentals of a refined numerical model adopted in this study for the determination of the flexural response, showing the consistency of this approach for the evaluation of the punching capacity of continuous slabs.

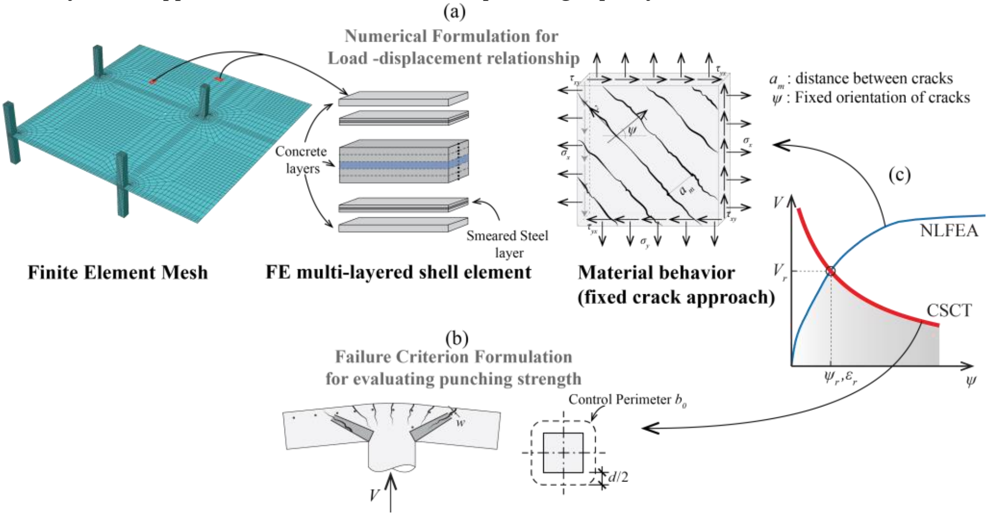

Fig. 2 Refined evaluation of the punching capacity (adapted from Cantone et al. (2016), [20]): (a) multi-layered element; (b) CSCT approach and control perimeter; and (c) calculation of failure load and deformation capacity.

In the framework of the revision of edge slab – column connection punching provisions for the new generation of Eurocodes, the goal of this paper is to investigate the structural behavior of continuous reinforced concrete slabs taking into account the nonlinear response of such members and the suitability of actual code provisions regarding the design of edge slab –column connections.

3 Determination of the flexural behaviour

Following the guidelines of MC2010, Level-of-Approximation IV (LoA IV) for punching design [3], a suitable load-rotation relationship in the vicinity of concentrated loads will be calculated by means of PARC_CL Crack Model, [21], Figure 2a. The numerical model PARC_CL is a constitutive model that can be implemented (by means of a user-subroutine in Abaqus) on a multilayered element to estimate a realistic response in flexure. Thick conventional 8-nodes shell elements with transverse shear flexibility (according to Mindlin Theory) and second order interpolation have been adopted for modelling slab member. The section is subvided in several layers with the aim to provide either pure concrete layers or concrete layers with smeared reinforcement along a prescribed direction, Figure 2a. As regards the mechanical behavior, PARC_CL Crack model is based on a fixed crack approach and smeared reinforcement. Regarding concrete and reinforcement constitutive laws, multiaxial state of stress and aggregate interlocking are taken into account as well as dowelling action, tension stiffening and residual tensile strength.

So far, several numerical investigations have been performed according to MC2010 (LoA IV) coupled with load-rotation curves determined by using PARC_CL. This approach has been adopted for different structural systems ranging from cantilever slabs subjected to concentrated loads (Natario et al. (2014), [22], Rombach et al. (2013), [23], Belletti et al. (2015), [24]) or to reinforced concrete members with irregular reinforcement arrangement and different column sizes (as tests by Ladner et al. (1977), [25], Cantone et al. (2016), [26]). The results (refer to Figure 3) confirm the robustness of this approach that appears to be suitable in the analysis of the flexural behavior of reinforced concrete slabs regardless of the boundary conditions, irregular reinforcement ratios or uneven slab geometries.

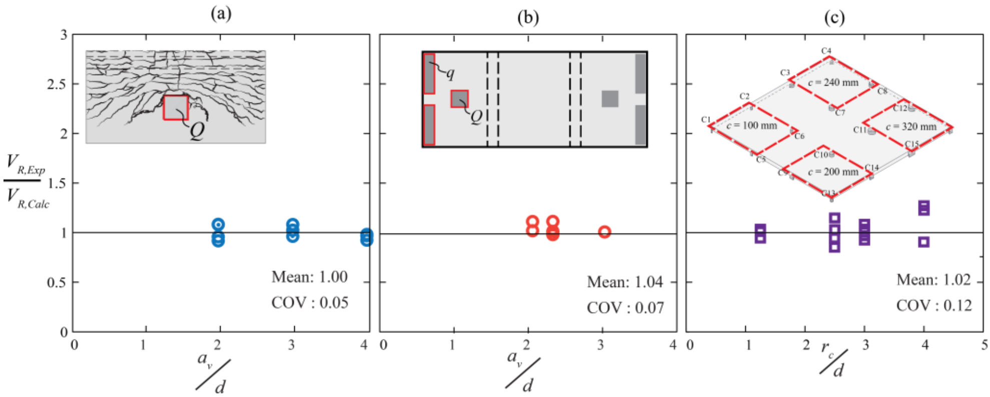

Fig. 3 Comparison of numerical model to the experimental results : (a) tests performed by Natario et al. (2014),[22] ; tests performed by Rombach et al. (2013), [23]; (c) tests on a continuous slab performed by Ladner (1977), [25].

For this reason, in the following, such approach will be used to investigate the structural behavior of edge slab-column connections as well as the main issues affecting their ultimate punching capacity, i.e.: the influence of loading eccentricity, moment redistributions and reinforcement arrangement.

For the calculation of the ultimate punching strength, the load–rotation response of the member has been calculated adopting the geometric average of the rotations in the direction perpendicular and parallel to the slab edge. This simplified consideration accounts for the fact that redistribution of shear forces may occur at the control perimeter and thus calculating the strength with the maximum rotation is conservative (a more detailed and general procedure within the frame of the CSCT for analyses accounting for the redistribution of shear forces within the control perimeter can be consulted elsewhere [13],[14]). This approach is validated, primarily, analysing isolated specimens PT33, PT34 provided, respectively, of a non-uniform reinforcement arrangement or uneven loading (refer to [13] for additional details), and, then, examining an experimental campaign on edge slab – column connections performed by Regan (1979) [10].

As it can be seen in Figure 4, the ultimate punching capacity calculated with the geometric average of the rotations is generally in sound agreement with the experimental punching resistance, even though the overall stiffness seems to be overestimated. This is also the case for the analysis of the test series perfomed by Regan (1979), [10], in which the calculated ultimate capacity agrees well with the experimental results (the eccentricity being dependent of the bending stiffness assumed for the column).

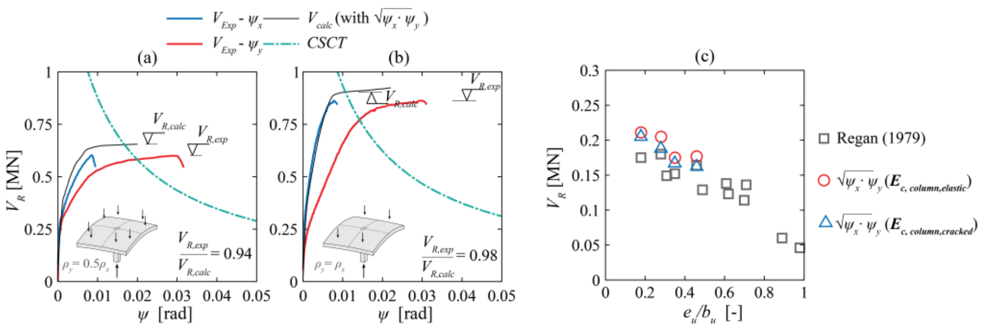

Fig. 4 Comparison of experimental and numerical load – rotation curves for (a) PT33 and (b) PT34; (c) comparison of experimatal ultimate punching capacity (tests by Regan (1979)) and calculated punching resistance (red circle: column stiffness considered in the elastic stage; blue triangle: column stiffness considered in the cracked stage, $E_{c,column,\ cracked}$ =4000 MPa).

In this paper, this approach is used to perform a parametric analysis on a continuous slab member. The edge slab – column connection has been discretized by modelling the slab with a multi-layered shell element approach (nonlinear constitutive laws according to PARC_CL Crack Model, [21]) while the column by means of solid elements maintained elastic in order to avoid material calibrations of solid elements after cracking (simulating uncracked columns subjected to high normal forces).

Figure 5 shows the case study selected for the parametric study on an actual continuous slab, focusing on the behavior of edge slab–column connections. Different reinforcement ratios have been adopted in the column and span regions in order to be comparable to reinforcement layouts of an actual continuous slab. The main parameters investigated were the influence of the top reinforcement perpendicular ($\rho_{hog,\,perp}$) and parallel ($\rho_{hog,\,par}$) to the slab edge as well as the ratio between the bottom sagging reinforcement and the top hogging reinforcement provided in the shear critical region at the edge slab column connection (ρ_{sag}, maintained uniform along the slab). It shoul be noted that elastic design refers to the case in which sagging reinforcement ratio ρ_{sag} is half the hogging reinforcement ratio while plastic design means to describe the case in which the ratio between sagging and hogging reinforcement is equal to one.

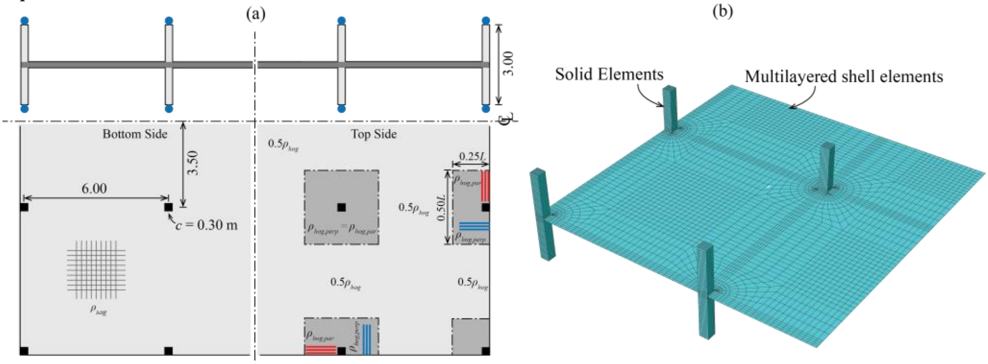

Fig. 5 (a) Reinforcement arrangement in hogging and sagging area for the case study underinvestigation; finite element mesh adopted for the numerical invetigation (edge span L_{edge}=6.00 m; central span $L_{central}$ = 7.00 m; column size c = 0.30 m; effective depth d = 0.21 m; concrete strength f_c = 35 MPa; yielding strength of the reinforcement f_y = 500 MPa).

4 Moment redistributions and influence of load eccentricity on the ultimate punching capacity

As mentioned in the previous section, several phenomena may influence the response of a continuous member with respect to its deformation capacity, resistance and distribution of internal forces.

So far, these aspects were mostly investigated for internal slab – column connections by several researchers, pointing out the presence of moment redistributions occurring between column and span region. Nevertheless, these redistributions may also occur in the shear-critical region of an edge slab–column connection, both in the perpendicular and parallel directions to the slab edge, Figure 6a.

Redistributions of bending moments shift the point of moment contraflexure affecting both the bending moment transferred to the column and the load eccentricity at the connection due to moment transfer, Figure 6. As experimentally observed by Regan, [10], after cracking in the hogging area, the eccentricity starts to decrease due to the difference between the flexural stiffness in the hogging and sagging region, Figure 6b. Nevertheless, once cracking occurs in the sagging area, the eccentricity at the connection increases again towards the elastic value, in particular for members with high hogging reinforcement ratios perpendicular to the edge (large bending stiffness after cracking).

Figure 6 shows the calculated eccentricities and the location of the point of moment contraflexure (r_s) in the case of an elastic design for four different hogging reinforcement ratio perpendicular to the edge. Nevertheless, it should be mentioned that the behavior after cracking of the sagging area is significantly influenced by the evaluation of the post-cracking bending stiffness provided by the constitutive laws adopted for the numerical investigation. Indeed, the potential overestimate of the reinforcement stiffness after cracking may cause an inaccurate estimate of bending moments leading to large values of the eccentricity. This aspect may be more pronounced in the case of an elastic design of the continuous member, in which the stiffness after cracking of the hogging region might be higher than the reinforcement stiffness in the sagging area.

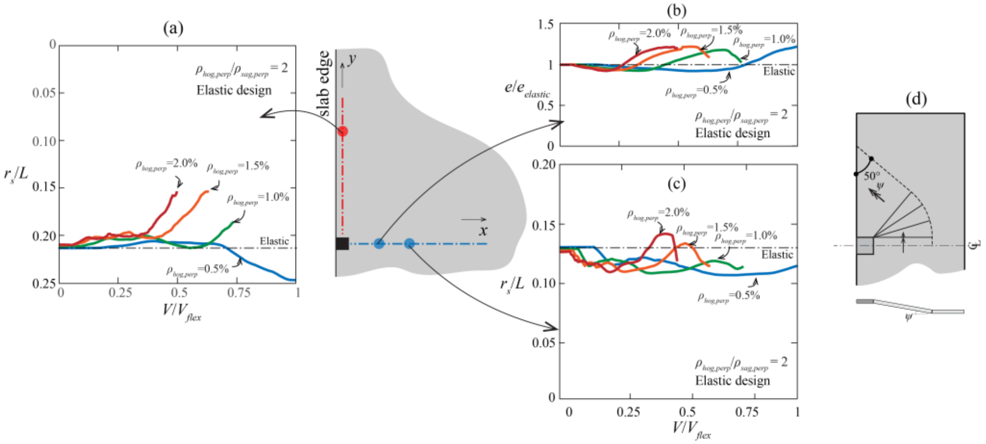

Fig. 6 (a) location of the point of moment contraflexure as a function of applied shear force V normalized by the flexural strength V_{flex} (in the direction parallel to the edge); (b) evolution of calculated eccentricity e with respect to the elastic value $e_{elastic}$ and (c) location of the point of moment contraflexure as a function of applied shear force V normalized by the flexural strength V_{flex} (in the direction perpendicular to the edge); (d) adopted yield mechanism for the calculation of the flexural strength.

Some relevant results of the analyses are presented in Figure 7. The evolution of the eccentricity at the edge slab–column connection according to an elastic and plastic design are shown in Figure 7 as well as the ultimate punching capacities corresponding to a member without and with shear reinforcement (cross and square labels distiguish two design types, i.e.elastic or plastic).

As it can be seen in Figure 7a, for members without shear reinforcement, the ultimate punching capacity is generally attained when the calculated eccentricity is lower than the elastic value. On the contrary, for shear-reinforced members, the calculated eccentricity might be higher than the elastic value, in particular for large hogging reinforcement ratios $\rho_{hog, perp}$, as shown in Figure 7a.

The ultimate punching capacity, as expected, increases for increasing reinforcement ratios, Figure 7b, in particular for members provided of shear reinforcement in which redistributions of bending moments may be larger than the case of members without shear reinforcement.

The role of sagging reinforcement perpendicular to the slab edge is observed to significantly influence the strength, particularly for members with shear reinforcement that fail at higher loads and deformation levels.

5 Conclusions

This paper reviews the approach to calculate the punching resistance based on the calculation of a refined load-rotation relationship (by using the PARC_CL Crack Model) and the failure criterion of the Critical Shear Crack Theory. This approach corresponds thus to an analysis of the punching strength according to a Level-of-Approximation IV by MC2010. By taking advantage of the numerical model of a flat slab describing its flexural behaviour, the redistributions of bending moments occurring at the edge slab–column connection have also been investigated. This investigation lead to the following conclusions :

I. Redistributions of bending moments perpendicular to the slab edge occur after cracking of the hogging area shifting the value of the eccentricity from the elastic threshold. After cracking of the sagging area, the ratio *M/V* starts to increase again towards the elastic value, shifting, in some cases, beyond it.

II. For members without shear reinforcement, the ultimate punching capacity is normally reached for eccentricities lower than the calculated eccentricity in the elastic phase. For slabs equipped with punching shear reinforcement (failing at higher loads and deformation levels), the eccentricity might however be larger.The sagging reinforcement may play a role on the ultimate punching capacity, in particular for shear – reinforced members.

III. The assumption of the geometric average for the evaluation of the ultimate punching capacity seems to be in sound agreement with the experimental results even though, occasionally, the capacity may be overestimated due to the calculated bending stiffness after cracking and the estimation of the column stiffness

Future developments will focus on the analysis of the reinforcement stiffness after cracking to better understand the consistency of the calculated eccentricity as well as on the distribution of internal forces around connections not presenting axis-symmetric conditions.

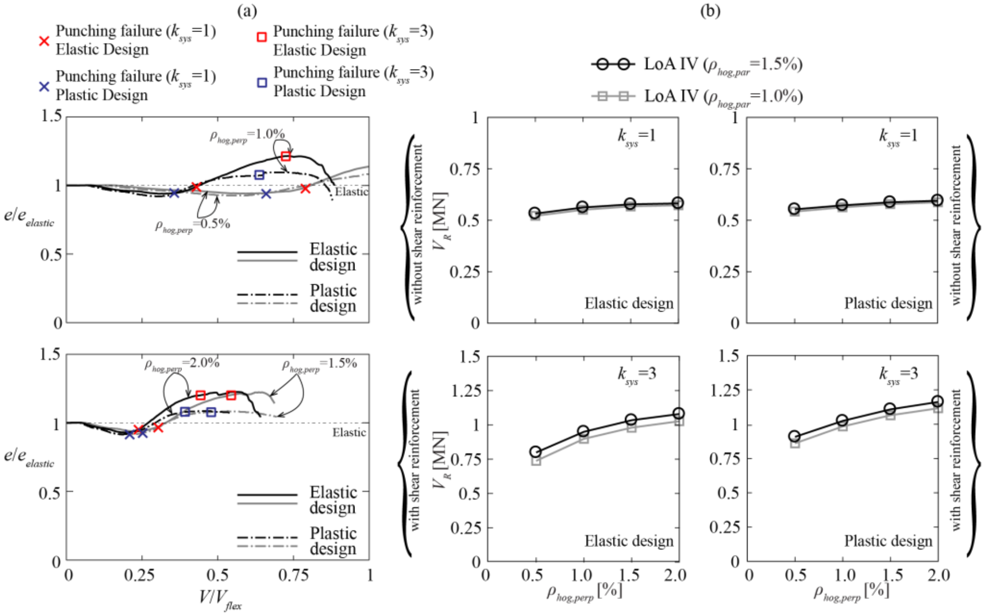

Fig. 7 (a) evolution of calculated eccentricity e with respect to the elastic value $e_{elastic}$ and corresponding ultimate punching capacity for members without (k_{sys}=1) and with shear reinforcement (k_{sys}=3); (b) ultimate punching capacity according to LoA IV for an elastic and plastic design.

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